NOTATION

A_l, solid-gas heat-transfer surface; V_l, volume of heat exchanger free of solid particles; M_l, mass of heat-exchanger solid; subscript l indicates that the quantity refers to a unit length of the heat exchanger; c_1, specific heat of solid; $\dot{m}$, mass flux of gas through heat exchanger; c_2, specific heat of gas; ρ_2, density of gas; T_1, temperature of solid; T_2, temperature of gas at point x; α, solid gas heat-transfer coefficient.

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MATHEMATICAL MODEL OF THE ELECTROCONTACT HEATING OF A STEEL BAR IN THE REGION OF CURRENT-CONDUCTING ELECTRODES

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A mathematical model of the electrocontact heating of a steel bar is developed, allowing the temperature fields in any cross section to be calculated. The adequacy of the model is verified by comparison with experiment.

The mathematical model of electrocontact heating (ECH) of steel bars proposed in [1] allows the temperature field to be calculated only in cross sections of the bar from its ends. In the present work, an attempt is made to develop this mathematical model of ECH so as to admit the possibility of calculating the temperature field in any cross section.

In cross sections far from the ends of the bar, axial heat currents may be neglected, and the temperature field is two-dimensional. Close to the ends of the bar, axial heat currents cannot be neglected. Therefore, it is necessary to solve the three-dimensional equation of heat conduction with internal heat sources

$$C_s D_s \frac{\partial t}{\partial \tau} = \lambda_s \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{\partial \lambda_s}{\partial x} \frac{\partial t}{\partial x} + \frac{\partial \lambda_s}{\partial y} \frac{\partial t}{\partial y} + \frac{\partial \lambda_s}{\partial z} \frac{\partial t}{\partial z} + q_V. \quad (1)$$

The heat transfer by convection and radiation occurring at the surface of the bar is taken into account as in [1]. Heat transfer between the heated bar and the water-cooled copper current-conducting electrodes is determined by two competing mechanisms: heat supply as a result of contact heat transfer and the liberation of additional heat in the region of the contacts because there is a transient electrical resistance between the contact electrode and the bar. In this case the boundary condition takes the form

$$-\lambda_s \frac{\partial t}{\partial n}\bigg|_{\text{sur}} + Q_C = \alpha_{CH}(t_{BS} - t_{CS}). \quad (2)$$

For contact heat transfer, α_{CH} is found from the formula [2]

$$\alpha_{CH} = 1.6 \cdot 10^4 \frac{\lambda_s \lambda_C}{\lambda_s + \lambda_C} \left(\frac{\bar{p}}{3\sigma_A} K \right)^{0.86} + \frac{\lambda_A}{\delta_E}. \quad (3)$$

Here K is a constant taking values from 1 to 3 depending on the roughness of the contacting surfaces. Since the plasticity of the copper is much higher than for steel, the thickness of the equivalent gas gap is [2]

$$\delta_E = 0.6 h_s. \quad (4)$$

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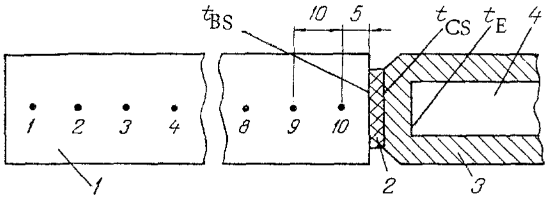

Fig. 1. Calculation scheme of ECH of experimental bar: 1) bar; 2) contact resistance; 3) current-conducting electrode; 4) cooling water.

The power liberated at the contact resistance and contributing to the heating of the bar is written in the form

$$Q_C = I^2 R_C k/S. \tag{5}$$

Here $k \leqslant 1$ is a coefficient determining what part of all the power liberated at the contact resistance contributed to the heating of the bar.

The electrical contact resistance when pure surfaces uncoated with oxide are in contact is determined mainly by the junction resistance and, according to [3], is

$$R_C = \frac{\rho_s + \rho_C}{8}\left(\frac{\pi \xi HB}{P}\right)^{1/2}. \tag{6}$$

Here ξ is the compression coefficient of the contact material (usually, $0.3 < \xi < 1$).

Equations (1)-(6) form the basis of the mathematical model of ECH in the region of the current-conducting electrodes. However, this system of equations is not complete, since it lacks expressions allowing the magnitude of the current passing through the bar (or electrode) and the distribution of heat sources to be calculated. These formulas must be taken from [1].

There is no analytical solution of Eq. (1). Therefore, it must be solved by numerical methods, e.g., by the finite-difference method.

The sequence of solution of Eq. (1), expressed in finite differences, is now outlined [4]. For simplicity, the one-dimensional case will be considered: the temperature distribution over the length of a thermotechnically thin rod at the ends of which current-conducting electrodes are clamped (one electrode at each end). Since the rod is symmetric with respect to its midpoint, it is sufficient to solve the problem for half its length.

The calculation scheme for this case is shown in Fig. 1. The left-hand boundary is assumed to be adiabatic; heat transfer at the bar—contact boundary is taken into account by introducing an additional fictitious half-layer of temperature

$$t_{N+1} = \frac{\left(1 - \dfrac{\alpha_{CH}\Delta x}{2\lambda_s}\right) t_N + \dfrac{\alpha_{CH}\Delta x}{\lambda_s} t_{CS} + \dfrac{\Delta x}{\lambda_s} Q_C}{1 + \dfrac{\alpha_{CH}\Delta x}{2\lambda_s}}. \tag{7}$$

In order to solve Eq. (7), it is necessary to know the temperature of the electrode surface. The following iterational procedure may be used for its calculation: 1) In the first stage, it is assumed that $t_{CS} = 20°C$; 2) t_{CS} is substituted into Eq. (7) and t_{N+1} calculated; 3) the next step is to find $t_{BS} = (t_N + t_{N+1})/2$; 4) the amount of heat entering the electrode is calculated from the formula

$$W_C = S\alpha_{CH}(t_{BS} - t_{CS}) + SQ_C(1-k)/k; \tag{8}$$

5) assuming that all the heat is removed by the water cooling the electrode, the temperature of the electrode wall on the cold side (t_E, Fig. 1) is found by the usual method; 6) the temperature drop in the copper wall of the electrode (Δt_C) creating a heat flux equal to W_C is calculated; 7) then $t_{CS} = t_E + \Delta t_c$ is found; 8) operations 2-7 are repeated until the difference in the values of t_{BS} found in two successive cycles is less than a specified amount (usually three to five iterations are sufficient).

The adequacy of the proposed method was verified by comparing the experimental and calculated heating curves of the experimental bar (the clearest picture is obtained in comparing the temperature drops over the length of the bar). Calculations were performed by an explicit finite-difference method of first-order accuracy [6] on a Minsk-32 computer.

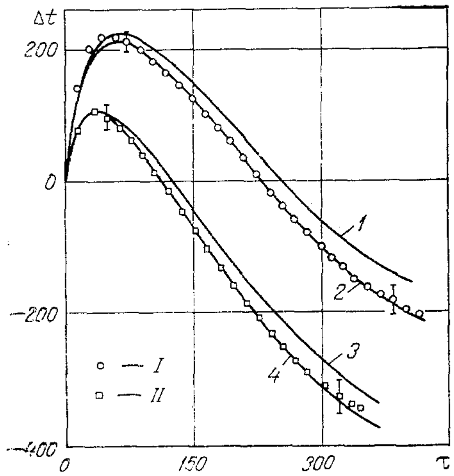

Fig. 2

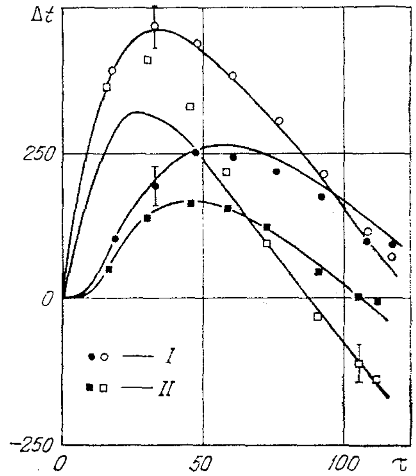

Fig. 3

Fig. 2. Temperature drop over the length of the experimental bar ($\Delta t = t_{10} - t_1$) at two values of the contact pressure. The mean current through the electrode is 1.8 kA. The continuous curves correspond to calculations: I) 390H, II) 1020H. Δt, °C; τ, sec.

Fig. 3. Temperature drop over the length of the experimental bar. The mean current through the electrodes is 2.8 kA. The open circles correspond to $\Delta t = t_{10} - t_1$ and the filled circles to $\Delta t = t_9 - t_1$. The continuous curves are the results of calculation: I) 390H; II) 1020H.

On the experimental apparatus described in [7], a bar of 12Kh18N9T steel of dimensions 20 × 20 × 300 mm was heated. In the course of heating, the temperature of ten points on the bar axis was measured and recorded using Chromel–Alumel thermocouples and a multipoint electronic potentiometer. The flow rate of cooling water and its temperature at the inlet to and outlet from the current-conducting electrode were also measured and recorded.

The bar was heated to temperatures of 800-1100°C in the middle section over its length, where the temperature is almost a linear function of the heating time. In the experiments and calculations, the current strength through the bar and the magnitude of the force pressing the contact electrode to the bar were varied.

The best agreement between all the experimental and calculated results was obtained under the condition that k = 1, i.e., that all the power liberated at the contact resistance contributed to heating of the bar. In addition, ξ has a value of 0.75-0.85, in good agreement with the values usually taken with mean purity of treatment of the contacting surfaces [3].

In Figs. 2 and 3, the calculated and experimentally measured graphs of the temperature differences at points 10 and 1 and 9 and 1 ($t_{10} - t_1$ and $t_9 - t_1$; point numbers as in Fig. 1) are shown. With constant contact pressure, the calculated curves, which coincide with the experimental curves in the initial section, subsequently diverge by an amount exceeding the experimental error (curves 1 and 3 in Fig. 2). This discrepancy arises because the contact pressure rises somewhat in the course of heating as a result of thermal expansion of the bar and unavoidable friction in the moving parts of the experimental apparatus, and this in turn leads to depression of the experimental curves.

In fact, including the rise in contact force at the end of heating by 17-20% of the initial value in the calculation, good agreement with experiment is obtained (curves 2 and 4 in Fig. 2). The calculated and experimental results are in very good agreement at values of the current through the electrodes of up to 3 kA (Fig. 3). Regrettably, we were unable to check the adequacy of the model at large values of the current through the electrode, although the need for such verification is obvious, since the limiting value of the current through the electrodes reaches 10 kA in real electrocontact equipment [8].

Nevertheless, the discrepancy between the calculated and experimental results should not be too large even at current values of the order of 3-10 kA. It may arise because we replaced the volume power liberated at the junction resistance by the surface power Q_C. This substitution may lead to change in temperature field in the junction region. However, for real cases, the length of the junction region does not exceed 10-15 mm according to our estimates.

Thus, it may be concluded that the proposed mathematical model of ECH, together with the model outlines in [1], allows the temperature field to be calculated with high accuracy in any cross section over the length of the bar, if the current through the electrodes does not exceed 3 kA. At large values of the current, quantitative results are valid for cross sections more than 10-15 mm away from the current-conducting electrode. The use of this model in designing powerful electrocontact equipment allows many constructional and operational parameters of the given equipment to be calculated.

NOTATION

C, specific heat; D, density; λ, thermal conductivity; ρ, electrical resistivity; t, temperature; τ, time; q_V, bulk density of internal heat sources; $\partial t/\partial n|_{sur}$, gradient of temperature field at the bar surface; α_{CH}, coefficient of contact heat transfer; P, force holding the electrode onto the bar; $\bar{p}$, specific contact pressure; S, geometric area of the electrode in direct contact with the bar; σ_A, ultimate strength of copper; HB, Brinell hardness of copper; h, mean height of microroughness; I, current passing through the current-condcuting electrode; R_C, electrical contact resistance; Δx, grid step. Subscripts: s, steel; c, copper; A, air; BS, bar surface; CS, contact surface; N, last half-layer of bar; N + 1, fictitious half-layer.

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